BREAKING IN THE SIMPLEX METHOD PART 1

Example 18. Given below is the simplex tableau for a maximization type of *LP* problem:

$C_j \rightarrow$	*Basic*	3	4	0	0	b_i
$\downarrow$	*Variable*	x_1	x_2	s_1	s_2	
4	x_2	1	1	1	0	6
0	s_2	1	0	−1	1	2

Answer the following questions with reasons:

(*i*) Does the tableau represent an optimal soultion?

(*ii*) Are there more than one optimal solution?

(*iii*) Is this solution degenerate?

(*iv*) Is this solution feasible?

(iv) Is this solution feasible?

(v) If s_1 is slack in machine A (in hours/week) and s_2 is slack in machine B (in hours/week), which of these machines is being used to the full capacity when producing according to this solution?

(vi) A customer would like to have one unit of product x_1 and is willing to pay in excess of the normal price in order to get it. How much should the price be increased in order to ensure no reduction in profit?

(vii) How many units of the two products x_1 and x_2 are being produced according to this solution and what is the total profit?

(viii) Machine A (associated with slack s_1, in hours/week) has to be shut down for repairs for 2 hours next week. How much will the reduction in profits be?

(ix) What is the maximum you would be prepared to pay for another hour (per week) of capacity each on machine A and machine B ?

(x) What are the shadow prices of the machine hours?

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$C_j \rightarrow$	Basic	Quantity	3	4	0	0
$\downarrow$	Variable		x_1	x_2	s_1	s_2
4	x_2	6	1	1	1	0
0	s_2	2	1	0	−1	1
	Z_j	24	4	4	4	0
	$C_j - Z_j$		−1	0	−4	0

- Does the tableau represent an optimal soultion?

- Yes. All Cj – Zj entries are zero or negative.

- Are there more than one optimal solution?

- No, it does not have more than one optimal solution because none of the non-basic variables(X1 and S1) has a zero Cj— Zj entry, in the final simplex table.

- Is this solution degenerate?

- No, because none of the basic variables(x2= 6 and s2= 2) has solution value equal to zero.

- Is this solution feasible?

- Yes, it is feasible because optimal solution is always feasible

- If s1 is slack in machine A (in hours/week) and s2 is slack in machine B (in hours/week), which of these machines is being used to the full capacity when producing according to this solution?

- Machine A is being used to the full capacity because value of slack variable s1 is zero in the optimal simplex table. See Basic variable SO SOLUTION X1=0 x2= 6 S1=0 and S2=2. s1=0 means full capacity on machine A. S2=2 MEANS, 2 HOURS ARE STILL UNUTILIZED ON MACHINE B

- A customer would like to have one unit of product x1 and is willing to pay in excess of the normal price in order to get it. How much should the price be increased in order to ensure no reduction in profit?

- The entry in the final Cj— Zj row under x1 is —1. This means the profit will be reduced by Rs1 if one unit of x, is added to the solution. Hence in order to ensure no reduction of profit, price of x, should be increased by rs 1.

- How many units of the two products x1 and x2 are being produced according to this solution and what is the total profit?

- x1 = 0, x2 = 6 and total profit is 24.

- Machine A (associated with slack s1 in hours/week) has to be shut down for repairs for 2 hours next week. How much will the reduction in profits be?

- Since marginal profitability of machine A is Rs 4 per hour, a reduction of 2 hours capacity in a week would cause a reduction of 2 x 4 = Rs 8 in profit. (see CJ – ZJ VALUE CORRESPONDING TO S1=-4)

- What is the maximum you would be prepared to pay for another hour (per week) of capacity each on machine A and machine B ?

- Machine A : 4 per hour; Machine B : nil.

- What are the shadow prices of the machine hours?

- Machine A : 4 per hour; Machine B : nil.