SIMPLE LINEAR REGRESSION Part 2

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Ex: The following data are given regarding expenditure on advertising and sales of a particular firm:

	Adv. Expenditure (X) (Rs. lakhs)	Sales (Y) (Rs. lakhs)
Mean	10	90
Standard deviation	3	12
Correlation coefficient	0.8	

- (i) Calculate the regression equation of Y on X.
- (ii) Estimate the advertisement expenditure required to attain a sales target of Rs. 120 lakhs.

- We are given,
- $\overline{X}$ = 10, $\overline{Y}$ = 90,σ_x = 3, σ_y = 12, r = 0.8

- Y depends on X
- Regression Equation of Y on X is:
- Y - $\overline{Y}$ = b_{YX} (X- $\overline{X}$)
- Y– 90 = 3.2(X-10)
- Y = 3.2X + 58

$$b_{YX} = r\frac{\sigma_Y}{\sigma_X}$$
$$= \frac{0.8 \times 12}{3}$$
$$= 3.2$$

- Regression equation of X on Y
- X – $\overline{X}$ = b_{xy} (Y – $\overline{Y}$)
- X – 10 = 0.2(Y –90)
- X = 0.2Y – 8

$$b_{XY} = r\frac{\sigma_X}{\sigma_Y}$$
$$= \frac{0.8 \times 3}{12}$$
$$= 0.2$$

- Since we have to find out advertisement expenditure (X) when Sales that is Y = 120 ∴ We will use regression equation X on Y
- $X = 0.2Y - 8$
  $= 0.2(120) - 8$
- = Rs. 16 lakhs

Ex: Given $\overline{X}$= 50, $\overline{Y}$ = 20,σ_x = 20, σ_y = 20 and Cov(X, Y) = -100. Find:

(i) Correlation coefficient
(ii) Both the regression coefficients

- $r = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y}$
- $= \frac{-100}{20 \times 20}$
- r = - 0.25
- (ii) The regression coefficients are:
- $b_{YX} = r \frac{\sigma_Y}{\sigma_X}$
- $= \frac{-0.25 \times 20}{20}$
- b_{YX} = - 0.25

- $b_{Xy} = r \frac{\sigma_Y}{\sigma_X}$
- $= - \frac{0.25 \times 20}{20}$
- $b_{xy} = - 0.25$

PROPERTIES OF REGRESSION COEFFICIENT

- **Property 1.** The coefficient of correlation(r) and the two regression coefficients (b_{XY} and b_{YX}) have the same signs.
- **Property 2.** The coefficient of correlation is the geometric mean between the regression coefficients.
- $r^2 = b_{XY} \times b_{YX}$
- **Property 3.** If one of the regression coefficients is greater than unity, the other must be less than unity. We know that correlation coefficient r ranges from -1 to +1 and therefore, $r^2 \leq 1$.
- Hence $b_{XY} \times b_{YX} = r^2 \leq 1$
- **Property 4**. The regression coefficients are independent of change of origin but not of scale.

- EX: The regression coefficient of regression equation of X on Y is 2.4 and the same for regression equation of Y on X is 0.8. Are the regression coefficients consistent?
- Regression coefficient of X on Y (b_{XY}) = 2.4 regression coefficient of Y on X (b_{YX}) = 0.8
- Therefore, as per Property 2:
- $r^2 = b_{XY} \times b_{YX}$
  $= (2.4) \times (0.8) = 1.92$
- Correlation coefficient r^2 is 1.92 which is greater than 1 and we know that
- $r^2 \leq 1$
- Therefore, it is not possible and thus the given regression coefficients are not consistent.